



**Geometric
Modeling
and Processing** 18TH



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Alternating Size Field Optimizing and Parameterization Domain CAD Model Remeshing

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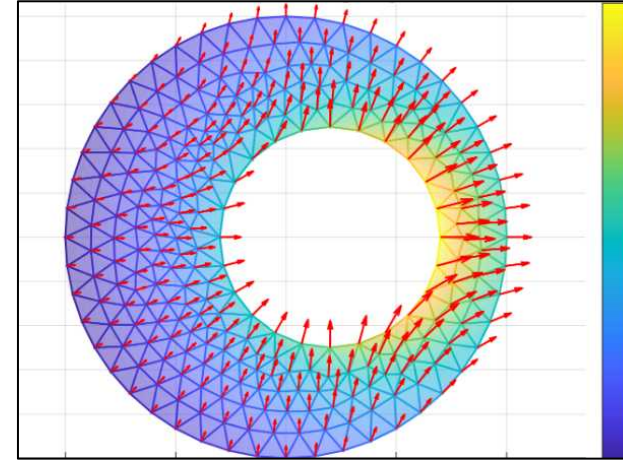
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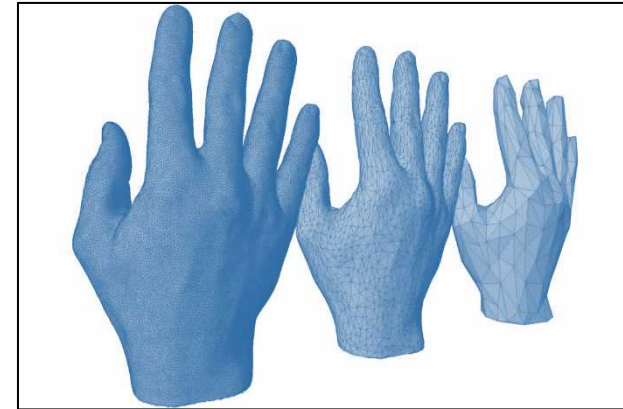
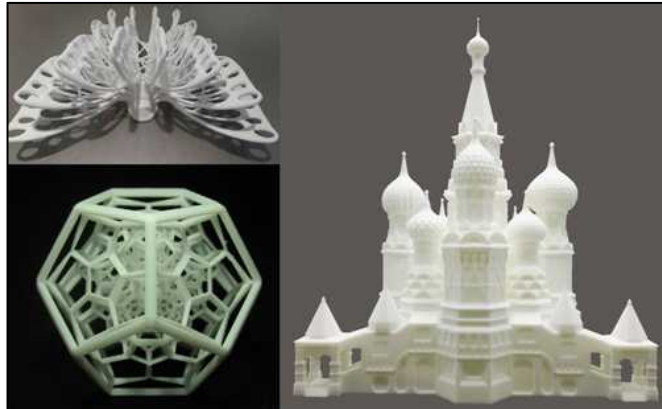
Why Triangulate CAD Models?

Rendering



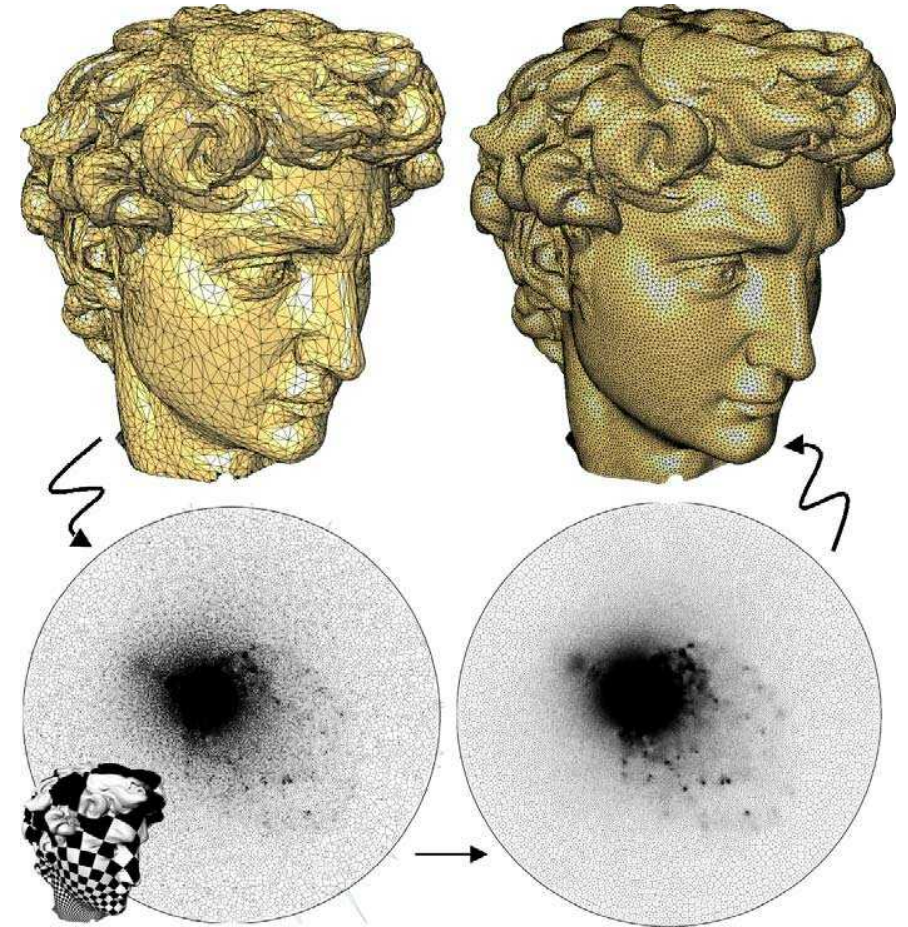
Finite
Element
Analysis (FEA)

3D Printing

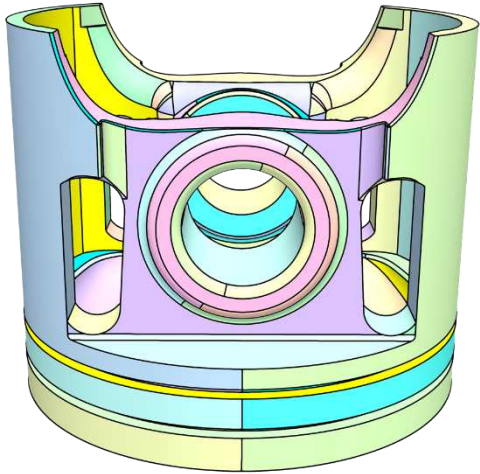


Mesh
Simplification

- Direct methods
 - CVT, ODT, OCT...
 - Remesh in 3D space
 - Projection, large computational cost!
- Parametric methods
 - Remesh in 2D space, and map into surface
 - Depend on parametric distortion
 - Optimality in 2D, but not 3D
- Hybrid methods
 - Parametric method + Direct method
 - Projection + Optimality issues



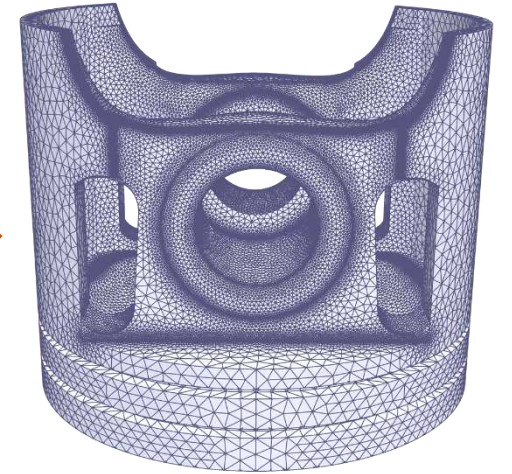
What is “good” Triangulation?



Input CAD model



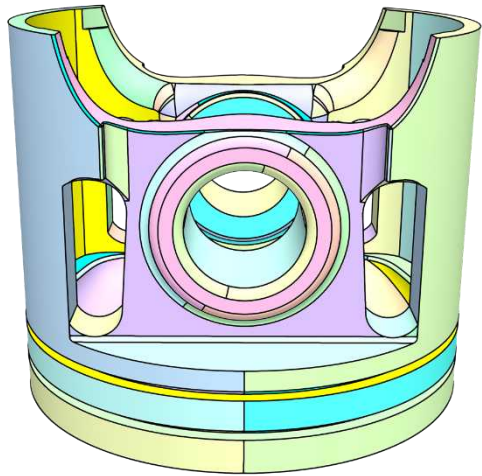
- Small number of elements
- Small Hausdorff error
- Good element quality
- Well-preserved features



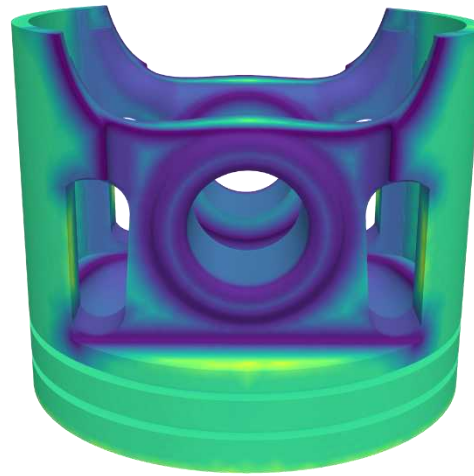
Output triangulation

How to formally describe these requirements?

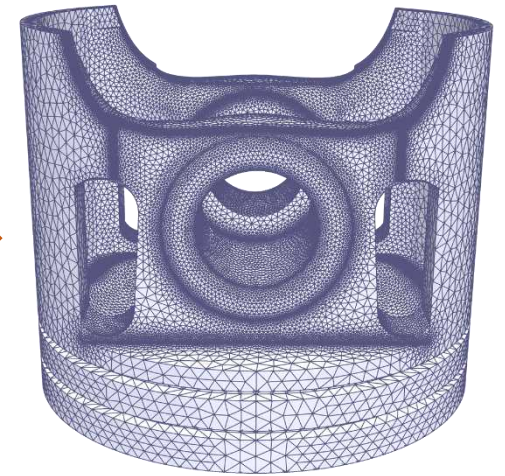
What is “good” Triangulation?



Input CAD model



Size field

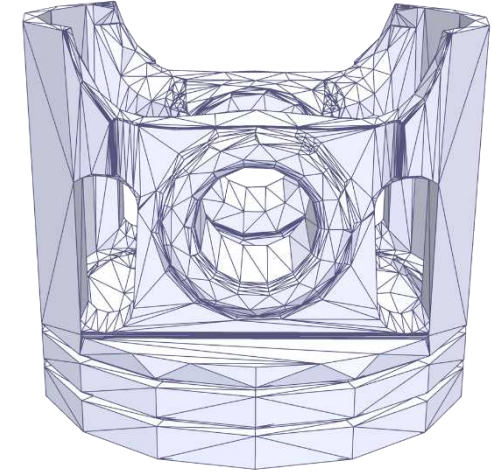


Output triangulation

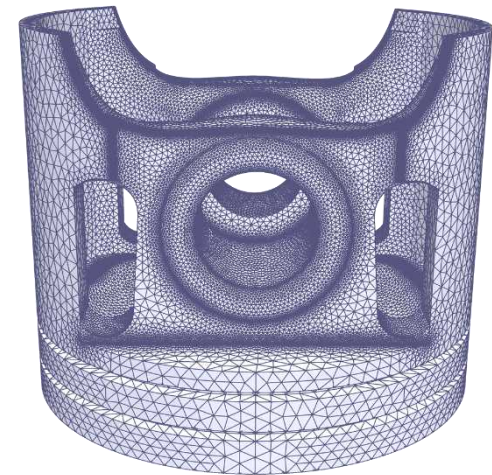
💡 **Uniformly describe them via a size field**

How to Discretize the Size Field?

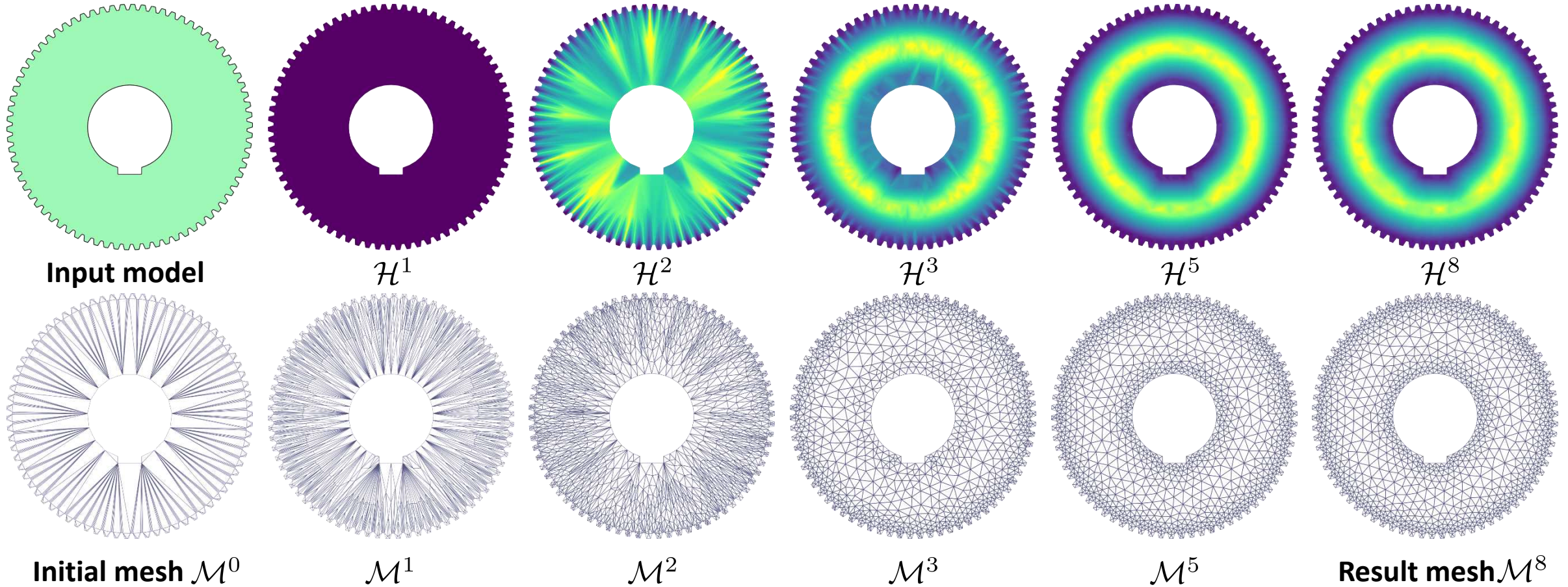
- Define on a background mesh
 - Intuitive and simple
 - Large computational cost 3D space query !
 - What is good background mesh?
Chicken-and-egg dilemma!



- 💡 Directly define on the current mesh
 - $O(1)$ query
 - Good triangle mesh = Good background mesh



Algorithm overview



Size Field Optimization

Size Field Optimization

Requirements of good triangulation

$$\begin{aligned} & \min \|\mathcal{H} - \mathcal{H}_{\max}\|^2, \\ \text{s.t. } & \|\nabla_t \mathcal{H}\| \leq \log(r), \quad \forall t \in \mathcal{T}, \\ & \mathcal{H}_{\min} \leq \mathcal{H}_v \leq \bar{\mathcal{H}}_v \quad \forall v \in \mathcal{V}, \\ & \mathcal{H}_f = \hat{\mathcal{H}}_f, \quad \forall f \in \mathcal{F}, \end{aligned}$$

Small number of elements

Good element quality

Small Hausdorff error

Well-preserved features

Constraints in the gradient form hard to satisfy!
The whole problem is also hard to convergence!

Size Field Optimization

Numerical strategies

- Relax to linear constraints

$$Q(x; a) = \begin{cases} 0, & x < a, \\ (x - a)^2, & x \geq a. \end{cases}$$

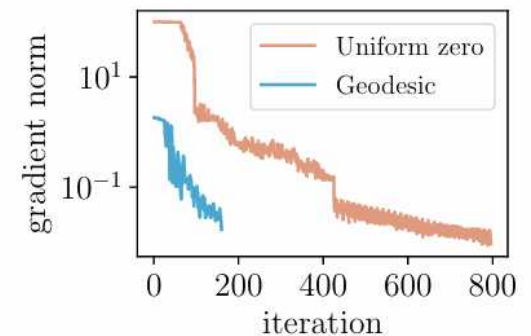
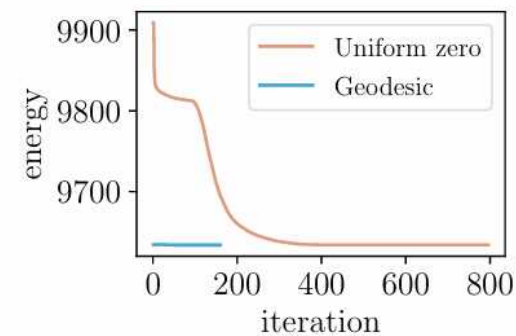
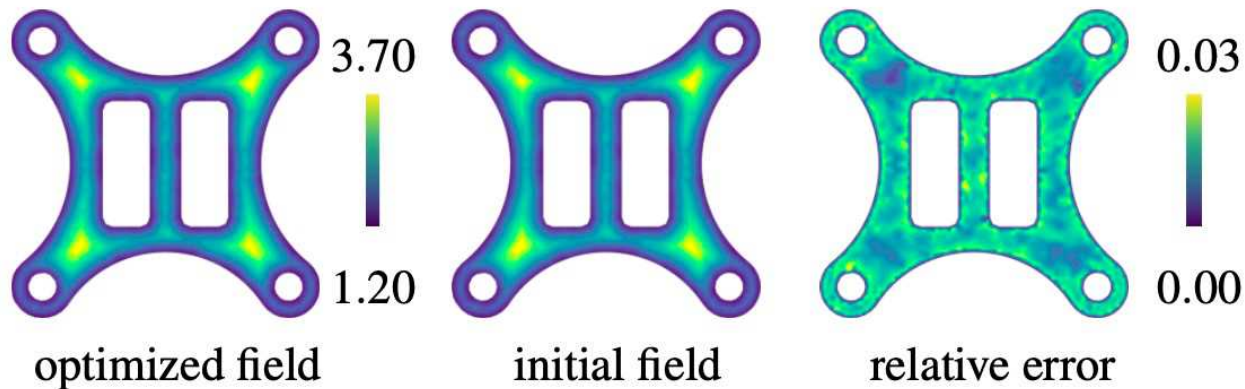
$$\min \|\mathcal{H} - \mathcal{H}_{\max}\|^2 + \sum_t w_t Q(\|\nabla_t \mathcal{H}\|; \log(r)),$$

$$\text{s.t. } \mathcal{H}_{\min} \leq \mathcal{H}_v \leq \bar{\mathcal{H}}_v \quad \forall v \in \mathcal{V},$$

$$\mathcal{H}_f = \min(\hat{\mathcal{H}}_f, \bar{\mathcal{H}}_f) \quad \forall f \in \mathcal{F},$$

- Initial value via geodesic

$$\mathcal{H}_n(p_n) = \frac{(r - 1)p_n + q}{r}.$$



Size Field Optimization

Numerical strategies

- Relax to linear constraints

$$Q(x; a) = \begin{cases} 0, & x < a, \\ (x - a)^2, & x \geq a. \end{cases}$$

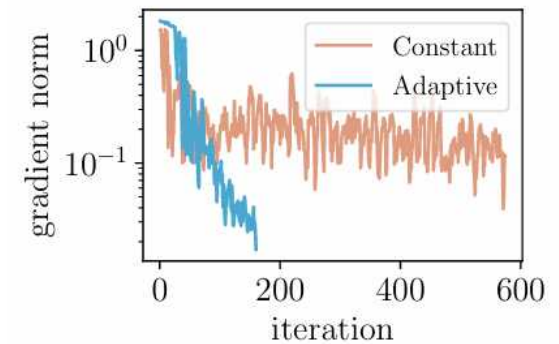
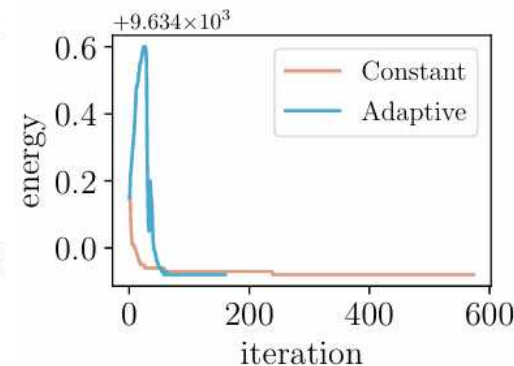
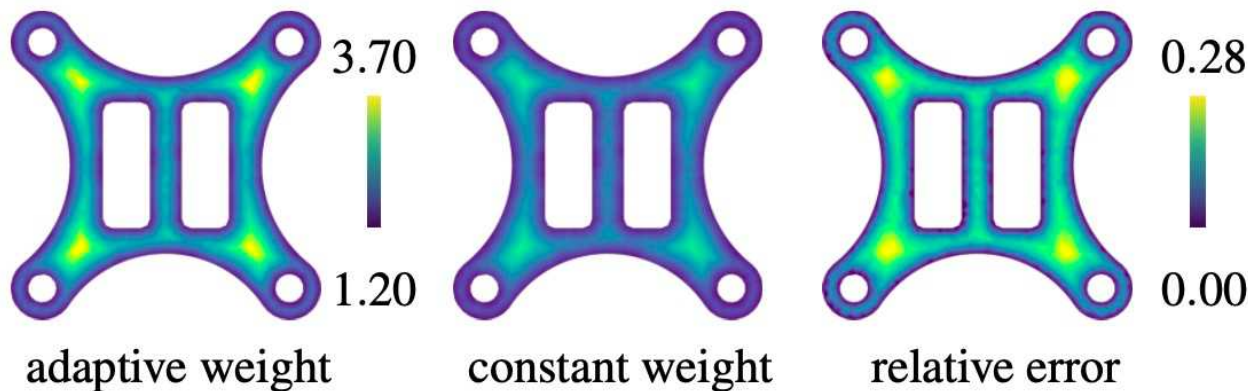
$$\min \|\mathcal{H} - \mathcal{H}_{\max}\|^2 + \sum_t w_t Q(\|\nabla_t \mathcal{H}\|; \log(r)),$$

$$\text{s.t. } \mathcal{H}_{\min} \leq \mathcal{H}_v \leq \bar{\mathcal{H}}_v \quad \forall v \in \mathcal{V},$$

$$\mathcal{H}_f = \min(\hat{\mathcal{H}}_f, \bar{\mathcal{H}}_f) \quad \forall f \in \mathcal{F},$$

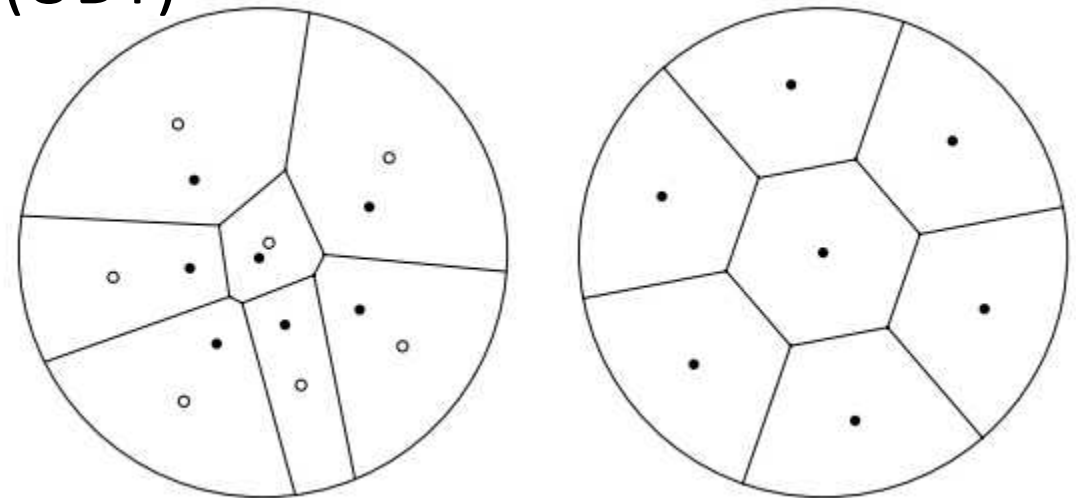
- Reweighting schedule

$$w_t^i = \begin{cases} \min(w_{\max}, w_t^{i-1} * 2), & \|\nabla_t \mathcal{H}\| > \log(r), \\ \max(w_{\min}, w_t^{i-1} / 2), & \text{otherwise.} \end{cases}$$



Geometry-aware Remeshing Operators

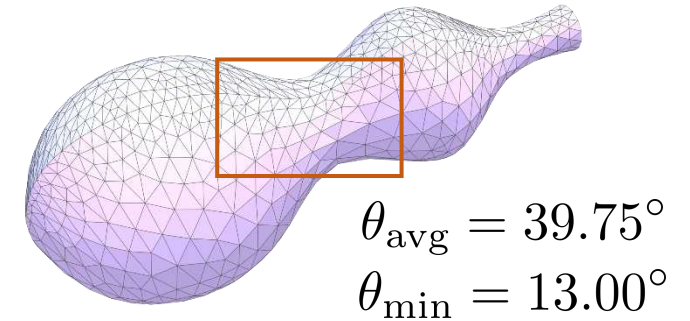
- Centroid Voronoi Tessellation (CVT)
 - Voronoi Cell $\Omega_i = \{\mathbf{x} \in \Omega \mid \|\mathbf{x} - \mathbf{x}_i\| \leq \|\mathbf{x} - \mathbf{x}_j\|, \forall j \neq i\}$
 - Site of the Voronoi cell $\mathbf{c}_i = \frac{\int_{\Omega_i} \rho(\mathbf{x}) \mathbf{x} d\sigma}{\int_{\Omega_i} \rho(\mathbf{x}) d\sigma}$
- Optimal Delaunay Triangulations (ODT)
 - Dual of CVT
 - ODT: vertices = CVT: sites



- ODT condition guided by the density field ρ

- Best location in the 3D space
- Not rely on the CAD surface

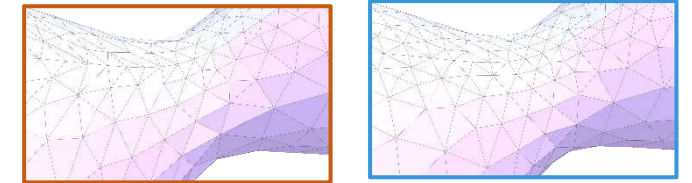
$$x^* = \frac{\int_{\phi(\mathcal{N})} x \rho(x) dx}{\int_{\phi(\mathcal{N})} \rho(x) dx}$$



- ODT condition in 2D and remap to 3D

- Always on the surface
- Optimality only in 2D

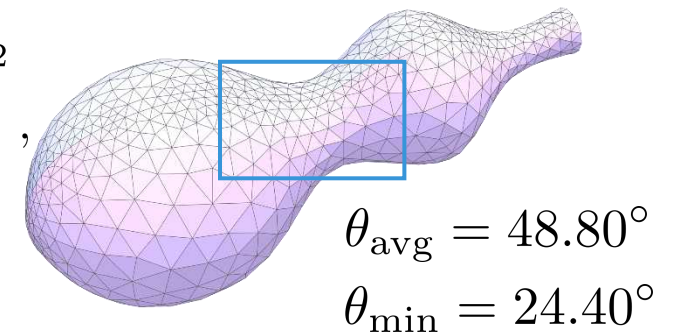
$$u^* = \frac{\int_{\mathcal{N}} u \rho(u) du}{\int_{\mathcal{N}} \rho(u) du}$$



- ODT in 3D + relocate in 2D

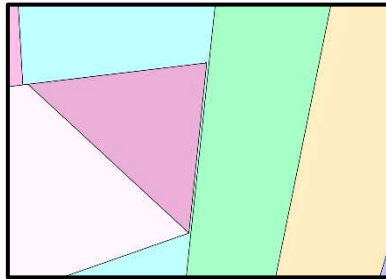
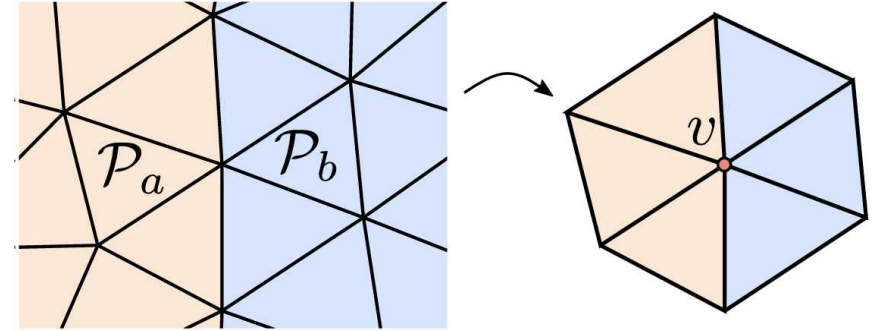
- Optimality + Intrinsic remeshing

$$\min_{u, x^*} \left\| x^* - \frac{\int_{\phi(\mathcal{N})} x \rho(x) dx}{\int_{\phi(\mathcal{N})} \rho(x) dx} \right\|^2, \quad \text{s.t.} \quad x^* = \phi(u), u \in \Omega.$$

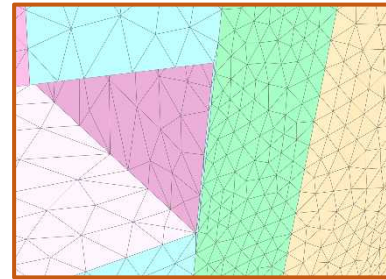


For a vertex shared by more than one patches

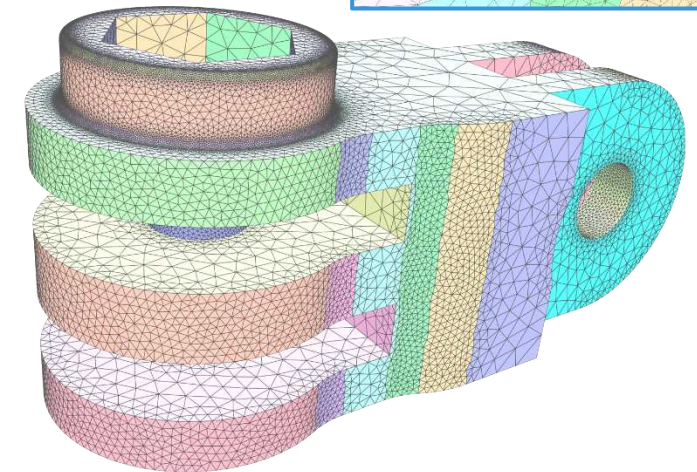
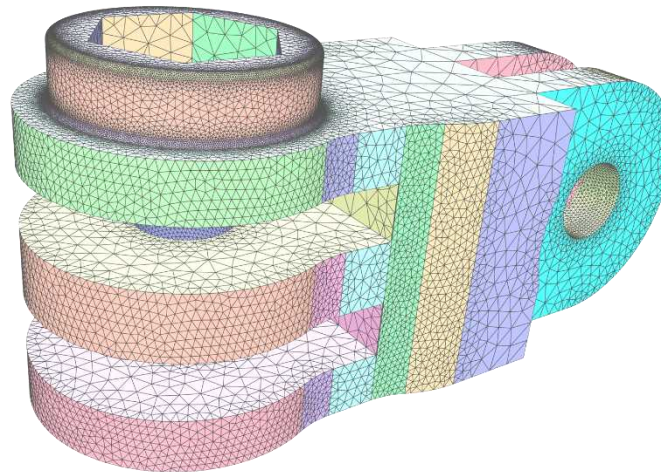
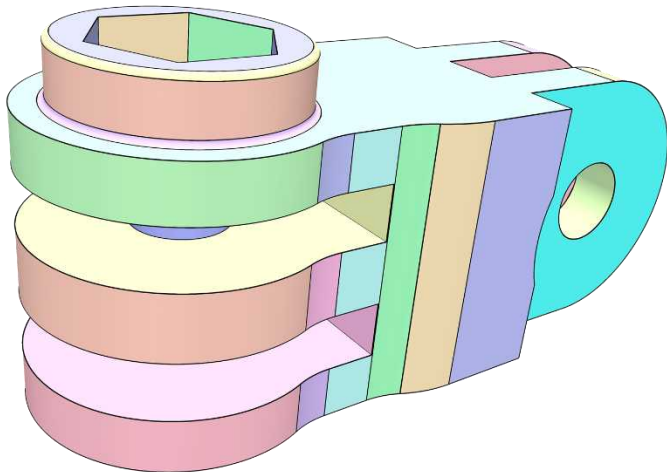
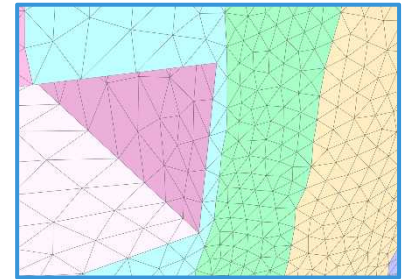
1. Find the optimal solution in each patch,
2. Select the solution with the smallest function value



$$\theta_{\text{avg}} = 48.98^\circ$$
$$\theta_{\text{min}} = 4.31^\circ$$

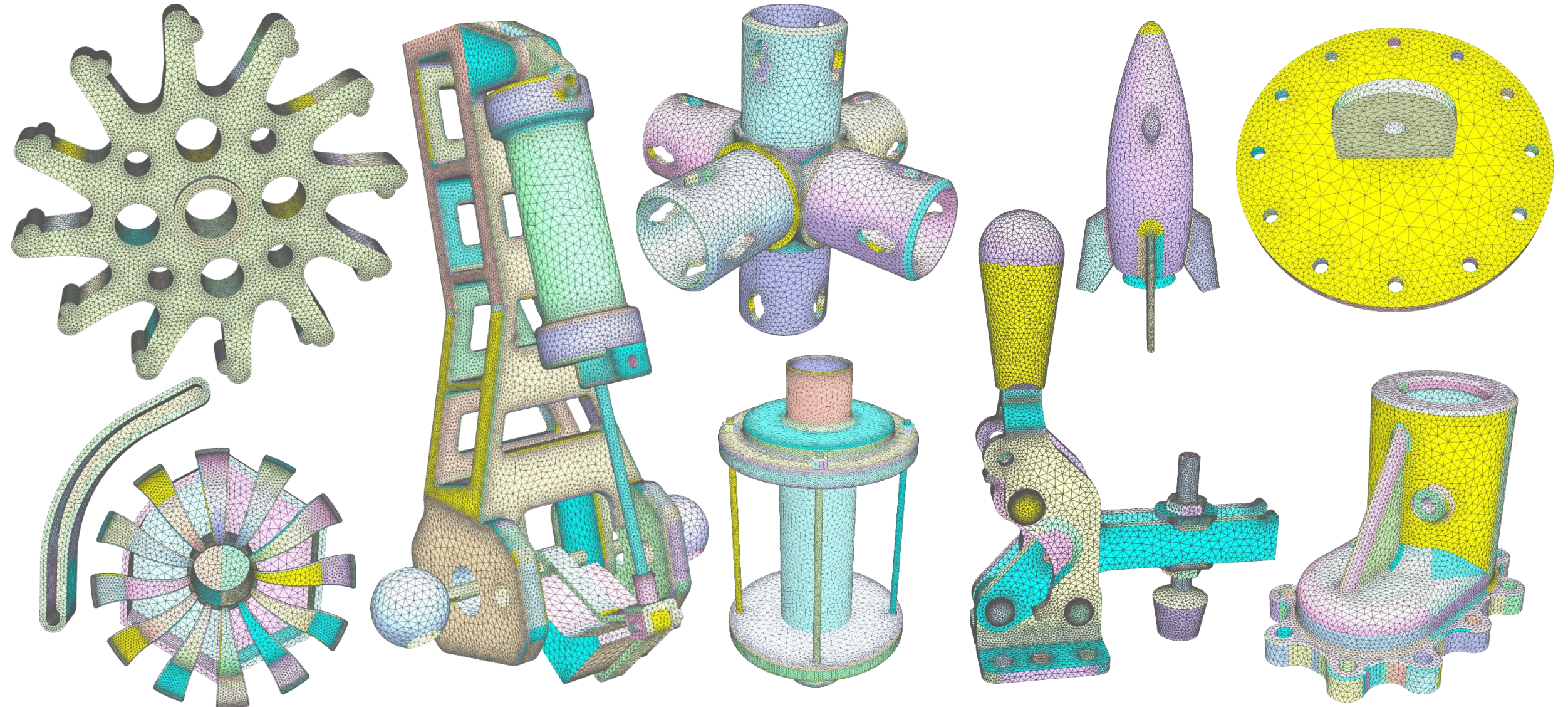


$$\theta_{\text{avg}} = 49.73^\circ$$
$$\theta_{\text{min}} = 23.98^\circ$$



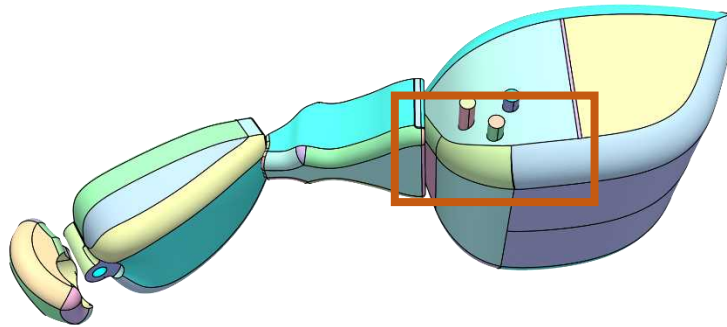
Results & Evaluations

Gallery of the Models

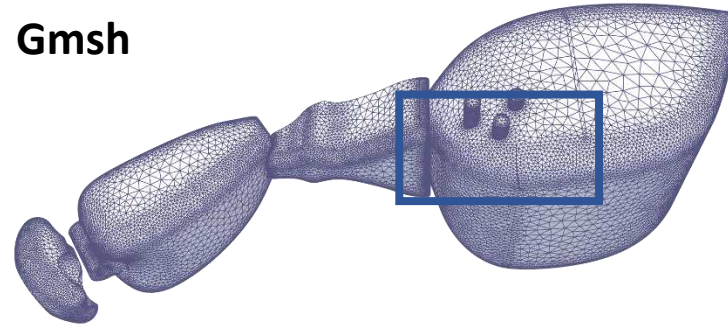


Comparison to Gmsh

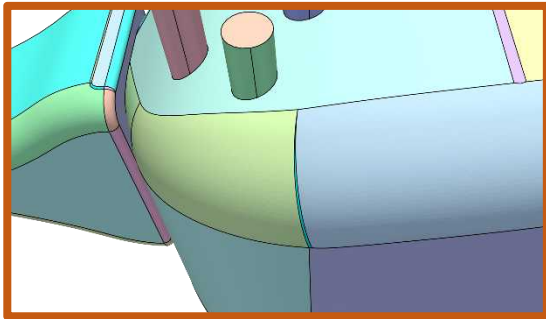
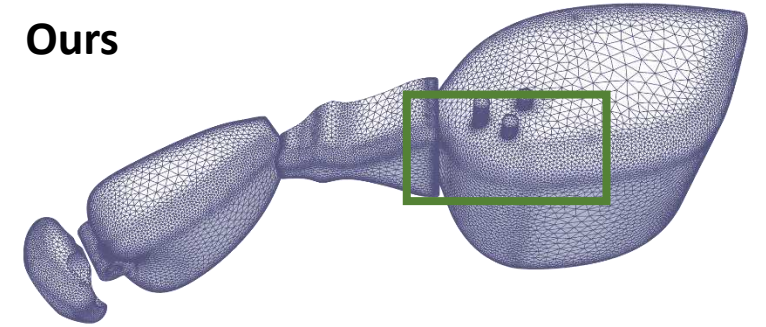
Larger minimal angles around the flat region.



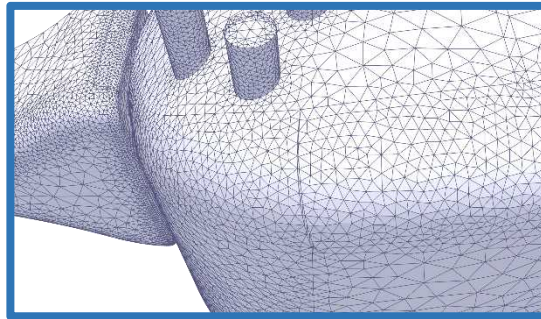
Gmsh



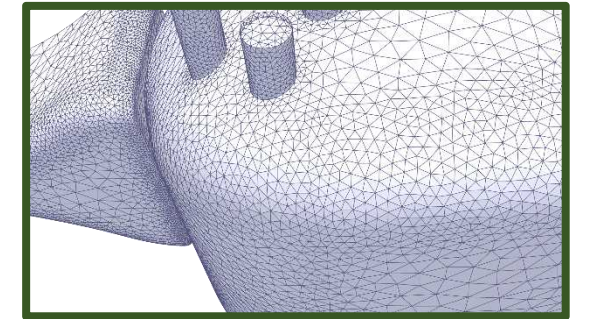
Ours



$V : 32044$
 $F : 64080$
 $\theta_{\min} = 1.69^\circ$
 $\theta_{\text{avg}} = 48.68^\circ$

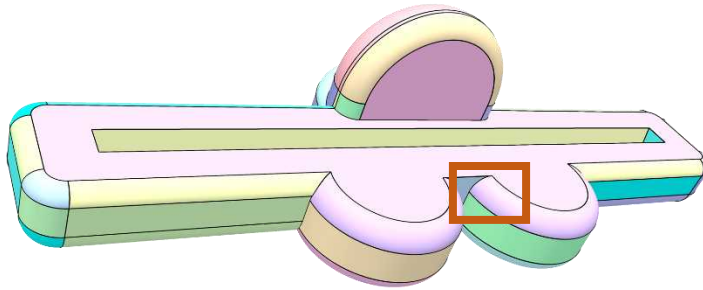


$V : 33180$
 $F : 66352$
 $\theta_{\min} = 6.01^\circ$
 $\theta_{\text{avg}} = 50.10^\circ$

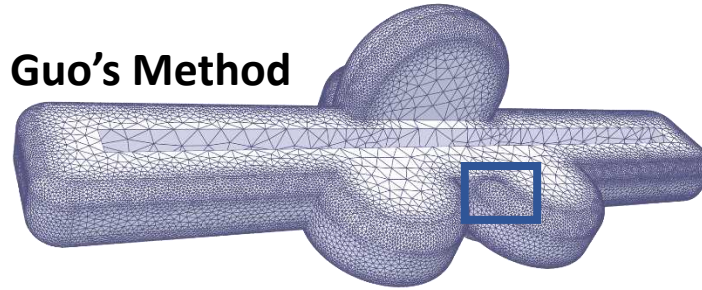


Comparison to State-of-the-art

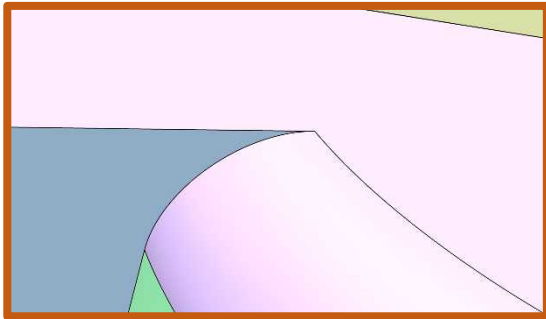
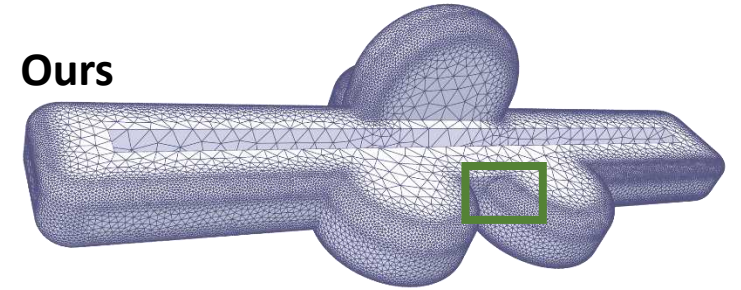
Projection free around the concave region.



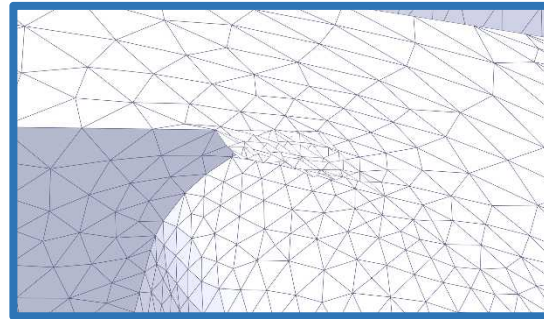
Guo's Method



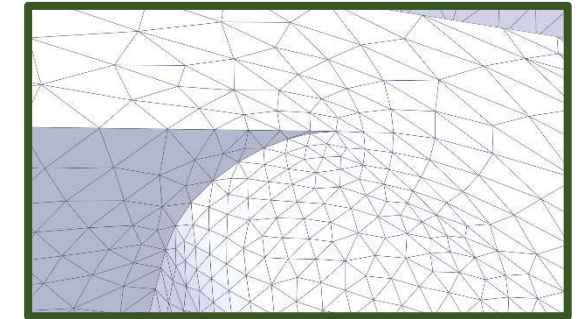
Ours



$V : 28030$
 $F : 56060$
 $\theta_{\min} = 3.83^\circ$
 $\theta_{\text{avg}} = 48.36^\circ$

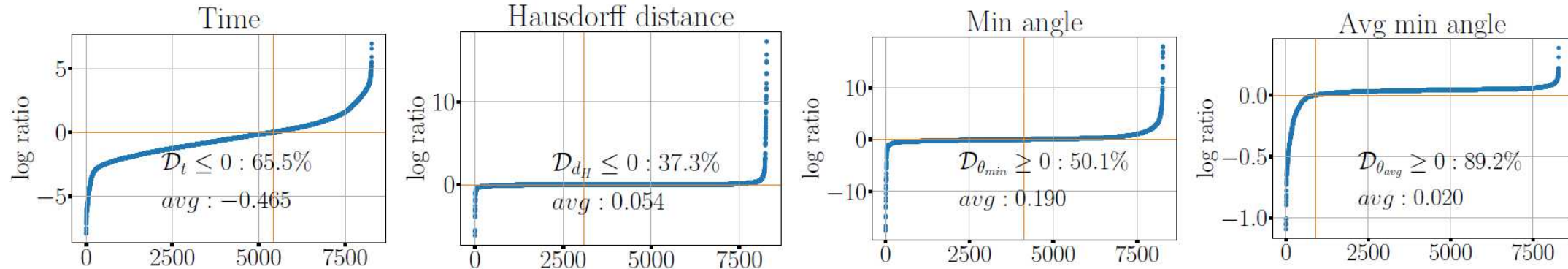


$V : 26875$
 $F : 53750$
 $\theta_{\min} = 5.20^\circ$
 $\theta_{\text{avg}} = 50.16^\circ$

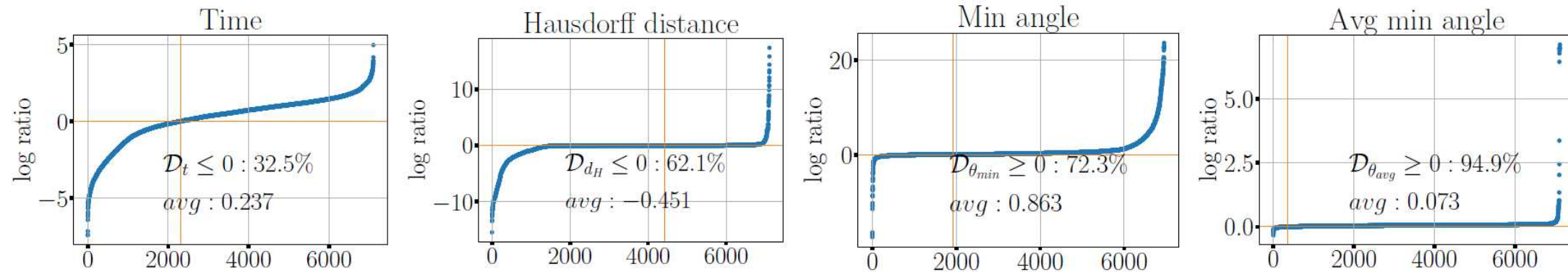


Distributions of comparisons

Distribution on comparison to Gmsh



Distribution on comparison to Guo's method



- Contributions
 - Alternative size field optimization and remeshing
 - Efficient initial value and adaptive re-weighting
 - Accurate approximation of surface ODT condition
- Limitations
 - Only consider isotropic size field
 - Cannot handle many singular cases in CAD models
 - Without theoretical guarantee to prevent degeneration



2024

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THANKS!

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